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ABSTRACT

The purpose of this paper is to demonstrate a solution to the annual problem of school district grade-wise enrollment projection. From the mathematical forecasting models considered, the exponential smoothing model is chosen and criteria for its evaluation specified. The model is then applied to data from three different school districts, and the results are found to correspond well with actual enrollments observed. A test against administrator judgment is made, and the model is extended to include trends. As a personnel and budgetary planning aid, the model appears to be well within error tolerance, cost-effective, and easily implementable. (Author)

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A [illegible] TO THE SCHOOL [illegible] PROBLEM[1]

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A SMOOTHING SOLUTION TO THE SCHOOL DISTRICT ENROLLMENT PROJECTION PROBLEM

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The purpose of this paper is to demonstrate a solution to the annual problem of school district grade-wise enrollment projection. From the mathematical forecasting models considered, the exponential smoothing model using ratios is chosen and criteria for its evaluation specified. The model is then applied to data from three different school districts and the results are found to correspond well with actual enrollments observed. A test against administrator judgment is made and the model is extended to include trends. As a personnel and budgetary planning aid, the model appears to be cost-effective, well within error tolerance, and easily implementable.

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A SMOOTHING SOLUTION TO THE SCHOOL DISTRICT ENROLLMENT PROJECTION PROBLEM

Characteristically, American school districts are faced with the difficulty of projecting annual grade-wise enrollments. The main purpose of this activity is to provide a rational basis for decision-making concerning personnel and budget preparation. Under conditions of population decline and migration among school districts, it becomes necessary to forecast accurately the number of students who will occupy each grade level since the ratio of students to teachers is relatively fixed and it is the teachers who must be hired, reallocated, or discharged. Hence, the enrollment projection problem must be solved with a minimal amount of error.

Possible Models

A number of forecasting models are available, some of which have already been applied to the projection of enrollment (1, 2).

Occasionally, forecasts are made strictly on some qualitative basis. This occurs under conditions of the failure of known models to project with sufficient accuracy, or simply a lack of knowledge of any mathematical models which might be applied to the problem. However, the application of a model usually is one in which either personal or group judgments enter into a "final" forecast. Such

qualitative methods are known as a <u>one-to-one</u> map of successful projection of school enrollments.

Markov process models as presented by Johnstone and Philip (3) along with cohort survival (Cortes, 4) provide the advantage of a system-wide view of student flow. Under relatively stable conditions of promotion within the system, such models have potential. However, both these models demand that individual students be tracked such that the school district information system must have on hand the particular history of movement of each pupil. Such knowledge is normally either not available or inaccessible, probably because the need for it has not heretofore been apparent.

The approach using indicator methods and multiple regression appears promising because once the key indicator variables and their time lags are determined, they may be used to predict enrollment change in an explanatory rather than simply a projective manner. This kind of model is, in a sense, independent of the past history of the enrollment process. The difficulty with multiple regression lies chiefly with the determination of the indicator variables (they tend to vary from district to district) and the high costs involved in the acquisition of relevant data.

A simulation approach involving many input variables and probabilistic outcomes is provided by Denham (5). The advantage is that forecasts which have their errors explicitly attached to them are more valuable to the educational decision-maker. Unfortunately,

the simulation model requires considerable data gathering, and the errors involved are relatively large.

The use of trend lines applied to actual enrollment counts is another method by which forecasts may be made. This procedure is especially appropriate when only prior knowledge of the enrollment for the previous year is at hand. A variation on the trend method usually referred to as classical time series analysis may be used to account for periodic fluctuations in enrollments that occur with monthly patterns within any given year, or for semester or quarterly patterns if applicable. Relatively sophisticated techniques such as that provided by Box and Jenkins (6) appear promising because of their usually high degree of forecast accuracy. However, the paucity of relevant observations coupled with a fairly high cost of implementation make such models comparatively unattractive at this time (7).

The remaining group of models is based upon a simple foundation known as ratios. This technique, which is described below, addresses only aggregates of persons enrolled in specific grades and both moving average and exponential smoothing models appear appropriate (8, 9).

Ratio Methods

In an attempt to predict grade-level enrollment from one year to the next, school district enrollment data may be presented in the form of a matrix where the rows represent divisions into grade-level and the columns are the September/June academic years. The

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problem is, of course, to forecast the column vector representing the number of students in the coming year. The most direct approach would be to utilize the simple time series within each row and arrive at a prediction for its next element. However, under usual conditions, students migrate from grade to grade each year as a group and hence the number of students one grade below during the present year becomes an important factor in the prediction of an accurate number of students in the following grade in the coming year. We are now focusing on apparent transition behavior of a group from one grade to the next. As the result of this, the ratio method resembles the Markov method in that transitions from one state to an adjacent one become a major concern. However, the ratio of these two numbers, R(t), (number of students in group j and grade i in time t divided by the number of students in group j and grade i-1 at time t-1) may exceed 1.0. This is because while there are invariably losses from the group during the interval, there are also gains in that other students usually join the group during the same time span. Ratios in excess of 1.0 are common when students are regularly sent from another school system into a specific grade level in the system under study. It is this phenomenon of the addition of students into the groups which makes the definition of the group vary from that of a strict cohort for which the membership would normally not be permitted to grow in size. Note that the model based on ratios does not render a forecast for the first grade. Unless the model is

applied to age groups of children in the district who are younger than their school-age counterparts, the information for a forecast is lacking. In such cases, a model may be applied directly to the enrollment data.

As defined, ratios are independent of the exact number of students occupying any particular grade at any particular time. Providing the problem of forecasting the ratios can be solved, then accurate predictions of next year's enrollments may be achieved. As was indicated, the two models most appropriate for this kind of analysis are moving average models and exponential smoothing. In order to choose between these two general kinds of models, it seems worthwhile to examine further some of the characteristics of student flow through school systems. Generally, an evolutionary style of change appears to take place over time. This pertains to shifts in student promotion-retention policies which tend to vary according to the prevailing educational philosophy, student leaving and returning behavior which is reflective of economic and social conditions, and student flow among institutions which varies according to local population migration and attractiveness of non-public institutions. Since enrollments vary as a function of such changing factors, the ratios vary as well.

For any given school system, ratio-data which are in excess of two or three years old may be considered somewhat irrelevant because of the accumulation of those previously mentioned changes. However, it is not

always possible to rely simply on last year's ratio since it in fact is subject to random error and hence is not a stable basis upon which to forecast the ratio for next year. This discussion leads us to a model which weights the most recent observations relatively heavily. It has been summarized by Geoffrion (10) and elaborated by Winters (11).

Exponential Smoothing

The ratios as defined above tend to exhibit a pattern which is relatively stable. Consequently, the following model is proposed:

$$R(t) = C + e(t) \qquad (1)$$

where R(t) is the ratio, C is a constant and where e(t) is a measure of error. The ratios will tend to vary about the same level over time, but the more recently observed ratios are more relevant to the accuracy of the projection. The basic smoothing equation may be written in a complete form:

$$\hat{R}(t+1) = aR(t) + a(1-a)R(t-1) + a(1-a)^2R(t-2) + \ldots + a(1-a)^nR(t-n) + \ldots + (1-a)^tR(0), \qquad (2)$$

where $\hat{R}(t-1)$ is the ratio to be used for the projection for next year for a specific grade. Each R( ) indicates the value of the ratio over successive years and the "a" is a constant to be determined empirically. Since the weights decline geometrically, observations of ratios in the distant past are given relatively little weight as compared to the most recent ratio, R(t). It may be shown that an

Iterative form of the same model is the following:

$$\hat{R}(t+1) = aR(t) + (1-a)\hat{R}(t-1). \qquad (3)$$

With both equations 2 and 3, the sum of the weights is equal to one. A projection of enrollments is made by multiplying the number of students in group j in grade i at time t $[X_{j,i}(t)]$ by the projected ratio $[\hat{R}(t+1)]$ to give the number of students in group j in grade i+1 at time t+1 $[\hat{X}_{j,\ i+1}(t+1)]$. Symbolically,

$$\hat{X}_{j,\ i+1}(t+1) = \hat{R}(t+1)\ X_{j,i}(t). \qquad (4)$$

Error Definition

Any attempt to apply an exponential smoothing model to real data brings forth two important considerations. One of these pertains to the treatment of error and for this there are at least three possible alternatives. For most school districts, allocations of teachers are normally not very flexible especially when teachers have rights to specific grade-level jobs through union contracts. Under these conditions, the crucial question becomes whether or not to hire or retain teachers for specific grade levels. The actual number of students overpredicted or underpredicted leads directly to a teacher equivalence via the student/teacher ratio which is usually constant. A reasonable error measure is the mean absolute value of the error, $\bar{E}_A$, across the grades:

$$\bar{E}_A = \frac{\Sigma|\hat{Y}-Y|}{n} \qquad (5)$$

where Y is the observed grade enrollment, $\hat{Y}$ is the grade enrollment projected from the previous year, and n is the number of grades.

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Such an error indicator is strictly dependent upon the size of each grade-level. If it were corrected by division by grade-level size to give the mean absolute value percentage error, then the degree of success of the model's application to enrollment projections in various school districts could be compared. This second method of error calculation may be expressed as:

$$\bar{E}_p = \frac{\sum\left(\frac{|\hat{y}-y|}{y}\right)}{n} \qquad (6)$$

A third approach to the error problem is based upon the calculation of the average error variance or mean square error (MSE) across grades. The square root of this average (the root mean square error or RMSE) is a measure of the average spread of error. This view of error has the further advantages of being the standard method of evaluation of the model and the one definition theoretically based since as Brown (9) points out, Equations 2 and 3 provide estimates which minimize the mean square error. It may be computed from:

$$MSE = \frac{\sum(\hat{y}-y)^2}{n} \qquad (7)$$

and the root mean square error, which is the standard deviation of the error distribution, found from

$$RMSE = \sqrt{\frac{\sum(\hat{y}-y)^2}{n}} \qquad (8)$$

Parameter Selection

The other problem in the application of the exponential smoothing model is the selection of the optimal value of "a". This becomes a matter of applying the model to data from past enrollments and examining the error as "a" varies across a chosen range of values. Since "a" may range over any value greater than 0 and less than or equal to 1, it seems reasonable to check each value of "a" across this range to determine the one associated with the minimum error. Naturally, the value "a" ultimately selected determines to a considerable extent the character of the model. Low values of "a" give relatively lighter weight to recently observed values of the ratios and relatively heavier weight to the groups of ratio values more distant in time. Such a projection is less sensitive to random "noise" and is thus

more stable. In contrast, a model in which the value of "a" is relatively high on the unit interval will allot considerable weight to recent ratio values and relatively little to the group of earlier values observed. Such a model is more responsive to recent changes and less stable overall.

The challenge in selecting a final and hopefully optimal value of "a" is parallel to the enrollment projection problem itself. Unless the system is extremely stable, the optimal value of "a" will vary year by year. The practical difficulty becomes the selection of the value of "a" which will minimize the error for a future forecast. Naturally, the basis for this decision rests upon the time series "a" associated with minimum error. Applying the same logic as above, it is possible to use a moving average based upon a number of recent observations of "a", or else to re-apply the exponential smoothing model weighing recent observations heavily.

Three Applications

Tables 1 to 3 present the root mean square errors averaged across grade-level as generated by the constant or simple exponential smoothing model applied to three different school districts. The first of these (Table 1) shows results for a large urban school district of declining enrollment. The data base for these errors

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Table 1 about here

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was the enrollments in all grades from 1973 back to 1958. The "a"

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values range from .05 to .95, with the minimum root mean square error falling between .20 to .95. The optimal value is .80 over the range of 7 years. The average RMSE (root mean square error) in the far right column shows that if an error of 10 students beyond the minimum is tolerable, then "a" may occupy any value between .45 and .95 inclusive. As may be seen from the table, 1972 was a year when the model performed markedly poorly. Further information reveals that it was a year of marked population shifts and considerable loss of students, most likely to suburban schools. The average of the root mean square errors for "a" = .80 is 96.10. Since the grade sizes were approximately 5,100 students, a tolerable amount of error was achieved. Underprediction would require unplanned hiring of new teachers, while overprediction would result in the redistribution of students among teachers with the constancy of the student/teacher ratio being maintained within a small margin.

A smaller, suburban district with decreasing enrollment was chosen for application of the model and the results are presented in Table 2. The range of the minimum "a" value is reduced somewhat, with the optimal value being .25, and for an additional error tolerance of ± 1 student, the value lies between .10 and .45 inclusive.

Table 2 about here

The root mean square errors are much smaller than for Table 1 since the district size is correspondingly reduced. The average RMSE for

"a" = .25 is 14.65. Again the size of the error is tolerable for important personnel decisions.

The third example (Table 3) is for another smaller suburban district, this time with increasing enrollments. For this district,

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Table 3 about here

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the optimal value of "a" is about .25, with an average MSE of 11.99. For a tolerance of ±1 student, the range of "a" is .15 to .45 and again, the error is within acceptable limits.

A comparison of the results for the three districts reveals that predictions for the large urban district are more optimal when weighted by higher parameter values, while those for the smaller suburban districts are better for lower parameter values resulting in a more uniform weighting. The high parameter value of .95 for some years for the large urban district suggests that the constant or simple exponential smoothing model may be less appropriate than one which incorporates trends. For an explanation and results of a double or trend exponential smoothing model, see the appendix. There is an absence of the usual finding that a greater number of total observations (in this case, children) tends to give more stable results. The range of the optimal value of the parameter is greater in the case of the large urban district than in the smaller suburban ones.

Table 4 presents a comparison between the results of the exponential smoothing model on the one hand and the best estimates of

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Table 4 about here

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the school district administrators on the other. The administrators' estimates were gleaned from a five-year moving average and corrected for reductions in grade-wise enrollments foreseen as a result of changes within that particular school district. Presumably, if the exponential smoothing model results were suitably adjusted, its error would be reduced still further.

Some General Conclusions

Some generalizations may be advanced on the basis of the three districts observed. One is that the RMSE is a curved function of "a", and that function is relatively flat near the optimal value for any given year and for all years taken together. Hence, it would seem that further division of the parameter between the .05 gradations would only slightly reduce the error. For all these functions, only one minimum exists when on the .05 scale.

Another conclusion is that there is no apparent trend in the optimal value of "a". This would suggest that the projection with least error should be based on a moving average of a small number of years (e.g. 5, 6, or 7, as was done in this study), or else a reapplication of the exponential smoothing technique. Since the margin of error for the parameter value is large, the moving average

rule is a simpler process which produces the same amount of accuracy.

A third generality which may be drawn is that whether enrollments are increasing or decreasing appears not to disturb the results of the model. This may be because some large changes may not result in the alteration of the ratios themselves, but may be felt through the original inputs into the student flow process: the entry into the first level.

Further, the optimal average root mean square error appears to be a concave, increasing function of district size. A further description must wait until the model is applied to more than three cases. Finally, the results of the trend or double exponential smoothing model (as demonstrated in the appendix) show an increased error over the constant or simple smoothing model. Unless trends in the ratios are pronounced and continuing, the constant model is preferable.

Commentary

This paper has presented only two kinds of enrollment projection applications of the exponential smoothing model. In a formal sense, the model which presumes a constant ratio (Equation 1) may be exchanged for ones which are more complex and to some extent more real. This implies the incorporation of trends into the data. While it was shown that the trend exponential smoothing model was less applicable to the present data, there is no question that marked trends often exist in the time series of ratios for school

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districts. Another possible variation to this technique is the projection of enrollments in excess of one year in advance. In this case, even a qualitative indication as to growth or shrinkage in grade size and hence school system size would be helpful to educational planners.

The application of the model may be advanced in a substantive manner by extending it to larger levels of grade-wise aggregation (e.g. state or province wide) for public school districts as well as other areas of education such as private, parochial, college, and university settings. It also appears applicable to groups of grades, except that the use of ratios then becomes misleading and a reversion to a simple time series would seem advisable.

It is quite possible that the greatest potential for this model may not lie in its extensions towards greater complexity. In its forms as presented in this paper (especially the constant or trendless version), the model is cost-effective, well within error tolerances, and has clear implications for resource allocation. It is also easily implementable. By "implementable" it is meant that school districts with their presently limited expertise and technology would be able to utilize such a model for their own planning purposes providing it is presented in a comprehensible manner.

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Table 1

Exponential Smoothing

Root Mean Square Errors for the Urban School District, Decreasing Enrollment[a,b]

"a" Value	1967	1968	1969	1970	1971	1972	1973	Average RMSE
.05	242.45	234.84	202.97	222.68	219.49	325.60	247.80	242.80
.10	160.91	164.59	127.02	147.50	165.28	274.91	189.00	175.61
.15	116.77	130.01	95.02	110.54	144.98	248.91	156.03	143.18
.20	94.73	116.21	87.70	94.35	137.09	233.89	135.38	128.60
.25	83.40	110.58	88.26	87.04	134.43	223.54	120.20	121.16
.30	76.74	107.43	91.40	82.73	131.31	215.40	107.75	116.10
.35	71.81	104.89	93.55	79.31	128.17	208.54	97.17	111.82
.40	67.80	102.51	95.21	76.17	125.22	202.61	88.30	108.26
.45	64.46	100.23	96.55	73.19	122.66	197.39	81.16	105.09
.50	61.75	98.05	97.76	70.33	120.60	192.75	75.84	102.44
.55	59.66	95.99	98.92	67.62	119.03	188.57	72.33	100.31
.60	58.19	94.00	100.09	65.05	117.91	184.74	70.72	98.67
.65	57.31	92.08	101.31	62.66	117.13	181.22	70.74	97.49
.70	56.97	90.21	102.57	60.45	116.61	177.94	72.21	96.70
.75	57.10	88.37	103.89	58.48	116.26	174.89	74.88	96.26
.80	57.66	86.56	105.26	56.77	115.98	172.02	78.49	96.10
.85	58.59	84.81	106.68	55.37	115.70	169.36	82.79	96.13
.90	59.86	83.15	108.15	54.34	115.37	166.91	87.62	96.48
.95	61.43	81.60	109.67	53.73	114.94	164.69	92.81	96.98

[a] 1973 enrollment was 60,303 [b] minima ± 10 underlined

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Table 2

Exponential Smoothing

Root Mean Square Errors for the Suburban School District, Decreasing Enrollment[a,b]

"a" Value	1968	1969	1970	1971	1972	1973	Average RMSE
.05	24.70	13.54	15.41	14.42	17.01	13.78	16.47
.10	23.90	12.32	13.99	13.77	16.09	13.40	15.57
.15	23.33	11.54	13.00	13.46	15.49	13.37	15.03
.20	22.91	11.17	12.35	13.35	15.10	13.50	14.74
.25	22.59	11.17	11.98	13.37	14.88	13.64	14.65
.30	22.33	11.45	11.82	13.45	14.75	14.39	14.69
.35	22.12	11.96	11.80	13.54	14.69	14.89	14.83
.40	21.95	12.63	11.88	13.62	14.69	15.[illegible]	15.03
.45	21.79	13.41	12.01	13.69	14.73	15.04	15.26
.50	21.66	14.27	12.18	13.73	14.81	16.46	15.99
.55	21.55	15.18	12.37	13.76	14.93	16.98	16.08
.60	21.46	16.12	12.57	13.78	15.09	17.49	16.08
.65	21.41	17.07	12.79	13.81	15.29	18.00	16.39
.70	21.38	18.03	13.02	13.86	15.52	18.50	16.71
.75	21.32	19.00	13.28	13.93	15.79	19.02	17.06
.80	21.44	19.96	13.56	14.04	16.10	19.54	17.44
.85	21.53	20.93	13.89	14.19	16.44	20.07	17.84
.90	21.66	21.89	14.27	14.40	16.81	20.62	18.27
.95	21.84	22.86	14.71	14.67	17.21	21.19	18.74

[a]1973 Enrollment was 5113. [b]minima ±1 underlined

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Table 3

Exponential Smoothing

Root Mean Square Errors for the Suburban School District, Increasing Enrollment[a,b]

"a" Value	1966	1967	1968	1969	1970	1971	1972	1973	Average RMSE
.05	23.13	16.38	13.32	16.47	12.12	12.40	11.15	13.67	14.84
.10	21.68	14.02	11.57	14.63	10.74	11.41	10.11	12.62	13.36
.15	19.96	14.35	10.58	13.36	10.10	10.89	9.55	11.82	12.40
.20	18.42	14.43	10.23	12.53	10.02	10.78	9.31	10.91	12.08
.25	17.60	14.92	10.30	12.03	10.27	10.84	9.36	10.52	11.98
.30	16.99	15.66	10.60	11.77	10.68	11.25	9.51	10.29	12.09
.35	16.54	16.50	10.99	11.65	11.14	11.63	9.72	10.17	12.29
.40	16.19	17.39	11.38	11.64	11.61	12.05	9.96	10.15	12.54
.45	15.93	18.28	11.74	11.70	12.04	12.47	10.17	10.19	12.81
.50	15.71	19.14	12.05	11.81	12.44	12.89	10.37	10.27	13.09
.55	15.55	19.98	12.30	11.97	12.81	13.31	10.56	10.39	13.35
.60	15.42	20.78	12.50	12.19	13.14	13.74	10.74	10.54	13.63
.65	15.33	21.56	12.69	12.45	13.46	14.18	10.91	10.72	13.91
.70	15.28	22.31	12.87	12.77	13.76	14.63	11.08	10.93	14.20
.75	15.27	23.04	13.06	13.14	14.07	15.11	11.26	11.13	14.51
.80	15.29	23.75	13.29	13.56	14.38	15.62	11.46	11.36	14.83
.85	15.36	24.46	13.59	14.03	14.72	16.16	11.68	11.59	15.19
.90	15.48	25.16	13.96	14.54	15.09	16.75	11.94	11.83	15.59
.95	15.64	25.86	14.44	15.11	15.51	17.37	12.24	12.07	16.03

[a]1973 Enrollment was 4016. [b]minima [illegible] underlined

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Table 4

A Comparison of the Results of the Exponential Smoothing Model with Administrator Judgement, 1973[a,b]

Grade	Exponential Smoothing	Error	Moving Average Plus Administrator Adjustment	Error	Actual Enrollments
K	253	18	235	0	235
1	260	8	244	24	268
2	280	14	289	5	294
3	316	10	311	5	306
4	350	16	340	6	334
5	380	16	383	19	364
6	379	23	373	17	356
7	362	1	364	3	361
8	393	4	392	3	389
9	382	4	382	4	378
10	416	3	421	2	419
11	394	4	398	10	398
12	368	12	375	5	380
RMSE	12.18		10.65		

[a] "a" = .65 [b] 1973 enrollment was 4483

APPENDIX

Because of the appearance of trends in the time series of some of the ratios, the exponential smoothing model was extended to include a trend component which was also smoothed. The equations for the extended model, referred to as "double exponential smoothing", are as follows: first, a projection is made according to the original model as in Equation 3 above.

$$S(t) = aR(t) + (1-a)\hat{R}(t-1), \qquad (A1)$$

where S(t) is equivalent to R(t+1) in Equation 3, which was the projection for simple exponential smoothing. Then a trend value is found:

$$B(t) = a[S(t)-S(t-1)] + (1-a)B(t-1) \qquad (A2)$$

where S(t-1) and B(t-1) are previous forecasts from Equations A1 and A2. A final projection is made with the following equation:

$$\hat{R}(t+1) = S(t) + (1/a)B(t) \qquad (A3)$$

where $\hat{R}(t+1)$ is now the ratio forecast from double exponential smoothing. Note that the trend value, B(t), receives considerable weight when "a" is small, and a weight of approximately 1 when "a" is large.

The results are presented in Tables 5 through 7.

Insert Tables 5, 6, and 7 about here

Comparisons between the pairs of Tables (1,5), (2,6), and (3,7) reveal an increase in the root mean square errors for the large, urban school district except for one year. Further, an increase is observed for all years for the small suburban district with decreasing enrollment, and for all years for the small suburban district with increasing

enrollment, except for one year.

The increase in average RMSE for all three cases suggests that the double exponential smoothing model is less appropriate to these data. On the surface of it, this finding may appear mysterious since the model makes the same forecast as for simple exponential smoothing when a trend is not present, as shown by Equation A3. An examination of the actual ratios reveals that while trends do exist for some grade levels in some school districts, these are short-lived and tend to disappear or reverse themselves. As a result, the model is "tricked" into following a trend and predicts accordingly, only to have the actual data behave erratically. Hence, the more naive simple exponential smoothing model results in a superior prediction for almost all years and systems.

The double exponential smoothing model does produce results which are more stable, however. This is shown by the smaller ranges of "optimal" "a" values for individual years. Fortunately, the optimal values are themselves closer together, producing a smaller range of overall optimal values as shown by the average RMSE columns. As with the simple exponential smoothing results, there appears to be no pattern or trend for the optimal parameter values. Consequently, the average RMSE value was used.

Table 5

Double Exponential Smoothing

Root Mean Square Error for the Urban School District, Decreasing Enrollment[a,b,c]

"a" Value	1967	1968	1969	1970	1971	1972	1973	Average RMSE
.05	2153.78	1995.47	1963.68	1871.31	1710.21	1592.19	1453.29	1818.84
.10	971.77	847.32	810.08	741.81	608.27	579.29	500.51	722.72
.15	566.02	463.19	440.54	384.75	275.18	307.59	233.21	381.49
.20	362.62	280.72	274.47	226.60	142.89	218.73	130.29	233.90
.25	248.80	183.15	193.28	149.46	97.19	189.22	88.55	163.81
.30	174.30	129.77	153.78	107.75	90.29	177.64	75.20	129.81
.35	127.28	101.13	135.41	85.74	94.70	171.31	76.35	113.13
.40	96.30	86.46	127.65	73.37	100.72	166.75	84.60	105.13
.45	76.38	78.17	125.22	66.31	106.30	162.91	96.09	101.76
.50	64.12	75.43	125.61	62.53	111.10	159.54	108.97	101.04
.55	57.34	73.27	127.56	61.12	115.12	156.59	122.38	101.91
.60	54.27	71.76	130.43	61.67	118.38	154.12	135.85	103.77
.65	53.50	70.57	133.84	64.01	120.70	152.24	149.13	106.28
.70	54.02	69.67	137.54	68.04	122.19	151.09	162.08	109.25
.75	55.18	69.16	141.36	73.70	122.77	150.78	174.63	112.51
.80	56.62	69.23	145.19	80.87	122.43	151.42	186.77	116.07
.85	58.16	70.12	148.97	89.45	121.24	153.08	198.56	119.94
.90	59.75	72.05	152.72	99.31	119.31	155.74	210.10	124.14
.95	61.43	75.21	156.40	110.33	116.85	159.35	221.56	128.73

[a] 1973 enrollment was 60,303 [b] minima ±10 underlined

[c] compare with Table 1

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Table 6

Double Exponential Smoothing

Root Mean Square Error for the Suburban School District, Decreasing Enrollment[a,b,c]

"a" Value	1968	1969	1970	1971	1972	1973	Average RMSE
.05	269.79	267.10	251.52	244.10	223.10	188.72	240.73
.10	139.47	119.96	106.11	101.69	84.48	67.56	101.37
.15	81.84	71.40	59.17	57.05	42.70	32.59	57.46
.20	59.12	47.79	37.13	36.87	25.49	19.87	37.71
.25	46.14	34.59	25.41	26.51	18.43	16.41	27.91
.30	38.13	27.11	19.17	20.97	16.14	16.56	23.01
.35	32.97	23.36	16.19	18.00	15.83	17.70	20.67
.40	29.54	22.17	15.07	16.42	16.24	19.00	19.74
.45	27.22	22.60	14.92	15.62	16.88	20.26	19.58
.50	25.60	23.94	15.23	15.28	17.63	21.47	19.85
.55	24.45	25.75	15.80	15.27	18.44	22.63	20.30
.60	23.61	27.79	16.56	15.53	19.29	23.79	21.09
.65	22.98	29.93	17.54	16.03	20.20	24.96	21.94
.70	22.51	32.10	18.79	16.74	21.16	26.18	22.91
.75	22.17	34.28	20.35	17.67	22.19	27.47	24.02
.80	21.94	36.46	22.26	18.80	23.30	28.84	25.23
.85	21.81	38.64	24.53	20.15	24.52	30.33	26.66
.90	21.79	40.86	27.17	21.75	25.88	31.95	28.23
.95	21.88	43.11	30.20	23.63	27.42	33.72	29.99

[a] 1973 enrollment was 5113.

[b] minima ±1 underlined

[c] compare with Table 2

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Table 7

Double Exponential Smoothing

Root Mean Square Error for the Suburban School District, Increasing Enrollment[a,b,c]

"a" Value	1966	1967	1968	1969	1970	1971	1972	1973	Average Error
.05	243.43	258.34	280.23	246.12	238.25	212.62	227.50	203.96	238.35
.10	116.30	111.58	119.43	94.06	88.57	71.57	79.03	66.09	106.80
.15	71.71	64.41	68.46	47.17	44.21	31.00	38.05	31.55	49.57
.20	50.07	42.43	44.74	26.29	26.02	15.26	22.47	19.85	30.89
.25	37.74	31.08	31.88	16.01	18.40	10.05	16.09	19.02	22.15
.30	30.14	25.56	24.46	11.57	15.75	9.76	13.52	14.75	18.13
.35	25.28	23.54	20.13	10.62	15.22	10.87	12.61	14.27	16.56
.40	22.11	23.50	17.67	11.30	15.43	12.19	12.43	14.07	16.08
.45	20.02	24.45	16.41	12.50	15.86	13.56	12.62	14.02	16.13
.50	18.60	25.87	15.95	13.82	16.35	14.98	13.00	14.00	16.58
.55	17.63	27.46	16.10	15.15	16.86	16.48	13.50	14.27	17.18
.60	16.93	29.09	16.74	16.47	17.42	18.05	14.08	14.58	17.92
.65	16.43	30.73	17.82	17.79	18.06	19.69	14.76	14.90	18.78
.70	16.07	32.35	19.33	19.12	18.82	21.38	15.57	15.51	19.76
.75	15.84	33.95	21.22	20.49	19.76	23.12	16.54	16.11	20.87
.80	15.65	35.55	23.50	21.94	20.93	24.93	17.71	16.79	22.11
.85	15.57	37.14	26.12	23.51	22.35	26.83	19.13	17.55	23.52
.90	15.58	38.76	29.09	25.29	24.06	28.86	20.85	18.40	25.11
.95	15.67	40.41	32.39	27.34	26.10	31.09	22.91	19.40	26.91

[a] 1973 enrollment was 4016

[b] minima ±1 underlined

[c] compare with Table 3

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